# **B&B** Training Associates Ltd

Certificate in Electrotechnical Technology

> City & Guilds 2330 Level 3

# Unit 1

Application of health and safety and electrical principles (Stage 3)

Outcome 4A

Understand the functions of electrical systems and components.

Learning Materials for the Electrical Industry This page is intentionally left blank

Produced by B&B Training Associates Ltd

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### Table of contents

Table of contents	5
Table of figures	6
Aims and objectives	7
Resistance and Ohm's Law	8
Series circuits	19
Series circuits	19
Parallel circuits	
Parallel circuits	27
Product over sum	
Power and energy	35
Energy	
Thermal energy	
Mechanical energy	
Electrical energy	
Power	44
Appendix 1	50

Table of figures	
Figure 1 Current flow around a circuit	8
Figure 2 Difference between emf and potential difference	9
Figure 3 Ohm's law triangle	10
Figure 4 Specific resistance of a material	12
Figure 5 Area differences	15
Figure 6 Resistors connected in series	19
Figure 7 Volt drop across resistors connected in series	20
Figure 8 Current in a series circuit	20
Figure 9 Parallel circuit	27
Figure 10 Current flow in a parallel circuit	28
Figure 11 Heat delivered to a substance	
Figure 12 Temperature rise in a current carrying conductor	
Figure 13 Potential energy	
Figure 14 Kinetic energy	40
Figure 15 Electrical energy converted to heat within a current carrying conductor	42
Figure 16 Low power output	44
Figure 17 High power output	

#### Aims and objectives

By the end of this study book you should be able to:

- describe resistance, what it is and what affects it
- state the relationship between resistance, voltage and current
- carry out calculations on series and parallel circuits
- carry out calculations on power and energy.

This study booklet covers part of Outcome 4 of Unit 1 of the new C&G 2330 Level 3 Certificate in Electrical Technology, it is broken down into sessions, which are aimed to last a minimum of 1.5hrs (one class). At the end of each session there will be a short assessment aimed at checking whether you have understood the subject matter. No answers will be provided for these questions and so you should make sure that you can understand the session before moving on to the next. In many instances, what you learn from one session will be added to in the next.

In many instances you will come across material that is very similar as you work through other outcomes. This is intentional. Too often people think that they understand a subject only to forget much of it when exam time comes around. By looking at similar areas in a variety of situations more knowledge should 'stick' as it is first learned and then applied.

Take your time and don't skip any areas just because you might be a little bored. At the end of the booklet there will be a longer assessment that covers the whole of the outcome. You should achieve at least 85 % in this assessment before moving on.

# Session

### Resistance and Ohm's Law

By the end of this session you will have had the opportunity to:

- understand resistance and its effects
- understand Ohm's law and apply it.

This part of the outcome 4 is very important and without a good grasp of this area, you will struggle in this course and in any future electrical work that you do. The principles gained here will stand you in good stead, so take the time to really study and grasp them now! I know this is revision, but you will need it.

Earlier, we mentioned that any electrical circuit requires three things, namely a source, a load and a means of transmission, and we have already looked at the definition of current, and seen that it can be simplified to '*a flow of electrons*'. The *source* provides, what is called, an *electromotive force* or *emf* to the circuit. This emf is the drive or the push that is applied to a circuit to force the current around the circuit.

### Symbol:-E

#### Unit:-volt (V)

Looking at Figure 1; if we assume that we are using conventional current flow, we can see that the source provides an emf to the circuit.





In this simple circuit we have a source (the cell), a load (that which we want to use) and transmission wires linking the source to the load. As we only have one load the value of the emf will be the same as the voltage that we would measure across the load.

When we measure the voltage dropped across the load we don't call this an emf but rather a **volt drop** or **potential difference**.

It may seem to be precise to highlight the differences but it is important. The emf provides the force that drives the current around the circuit, this circuit may have any number of loads connected to it, at which point the emf would not be the same as the voltage dropped but would be made up of all the voltages dropped across the loads.

In Figure 2 we can see that the voltages dropped across each resistor are different to the emf. When they are combined however, they do add up to the same value as the source emf.



Figure 2 Difference between emf and potential difference

Although emf and volt drop or potential difference, are measured in volts, the symbol is different, and it is important that you start to use the correct terminology. You must start to talk the language.

#### Symbol:-U or V Unit:-volt (V)

You will use **U** more than you will use **E**. **E** is used when we are looking at the voltage source such as a battery, whilst **U** is used to describe the volt drop or the potential difference. You may come across books and people who still use **V**. This is not a great surprise as **V** has been used for a long time.

We come now to the most important formula that you are going to come across. It is simple and yet is used in so many seemingly different situations. It is called **Ohm's Law**.

Georg Simon Ohm in 1827 discovered that if you have a metal, maintained at a constant temperature for a constant length and area, and then if you increased the voltage the current increased in the same proportion.

Simply put, he stated that voltage was proportional to current or  $U \propto I - (\infty)$  means proportional – to when the temperature of a metal was held constant. He also wondered what it was that linked the voltage and the current, and this link was the **resistance** of the conductor.

#### Resistance is the opposition to current flow.

If you can picture in your mind's eye a hosepipe, resistance would occur when you squeezed the pipe, the more you squeezed the greater the opposition to the flow of water, and this is similar to the idea of electrical resistance.

Ohm's Law can be remembered in a number of more useful formats and the most common are:-

$$U = IR \quad I = \frac{U}{R} \quad R = \frac{U}{I}$$

A triangle similar to the one used for **quantity of electricity** can be of use for Ohm's Law. Try to transpose, but if you can't use the triangle



Figure 3 Ohm's law triangle

It is essential that you remember Ohm's Law, and are confident enough to use it in a wide variety of situations. Follow these examples.

1/. The insulation resistance between a line conductor and earth is 0.5 MΩ. If the supply voltage is 230 V what will be the leakage current?

Remember to be careful how you '**unpack**' the question. Don't get bogged down by some of the terminology.

$$U = IR \text{ transpose}$$
$$I = \frac{U}{R}$$
$$I = \frac{230}{0.5 \times 10^6}$$
$$I = \underline{460 \, \mu A} = \underline{0.46 \, m A}$$

2/. A current of 40 mA flows in a circuit through a resistance of 18 k $\Omega$ . What is the voltage dropped across the resistor?

Again, do not get confused by the terms used. You should already be familiar with volt drop, and recognise that all that is being asked for is the voltage.

$$U = IR$$
$$U = 40 \times 10^{-3} \times 18 \times 10^{3}$$
$$U = \underline{720V}$$

The principles are straightforward, but still mistakes can be made; so be careful!

Again, don't worry that you have looked at this before, this is not time wasted, as it is rare for people to get the principle the first time round.

When any material has a supply connected to it, it has a particular resistance. This particular resistance is called the '*resistivity*' of the material or its '*specific resistance*'.



Figure 4 Specific resistance of a material

Material	Resistivity at 0°C Resistivity at 2	
	$ imes$ 10 <sup>-8</sup> $\Omega m$	$ imes$ 10 <sup>-8</sup> $\Omega m$
Copper	1.588	1.724
Aluminium	2.6	2.83
Brass	6.02	6.2
Lead	20.3	22.0
Mercury	95.1	96.8
Silver	1.5	1.62
Tungsten	4.96	5.51
Nichrome	108	108.2
Iron	8.81	10.1

Below is a table of many of the relevant values of resistivity for common materials.

Once the temperature varies then the resistance values vary, and so any figure needs to be quoted at a certain temperature.

Looking at resistance, what we have is a situation where if you have a particular material it will have a particular **specific resistance**. If the length of that material is increased, it will also change its resistance, in fact increase the length and you increase the resistance by that same proportion. If that same material changes in size, or area, then it will change its resistance.

Increasing the area reduces the resistance.

What we now have is the ability to formulate an equation that can be used.

(*rho*) is the symbol used for resistivity

$$R = \frac{\rho I}{A}$$
  
Where  $R = \text{resistance}(\Omega)$   
 $\rho = \text{resistivity}(\Omega m)$   
 $I = \text{length}(m)$   
 $A = \text{area}(m^2)$ 

#### Remember:-

# Increasing the length increases the resistance. Increasing the area decreases the resistance

The general rule that you need to remember when working out any problem with regard to resistivity is to make sure that the units that you use are consistent. What this means is that if an area is given in mm<sup>2</sup>, then the length must also be in mm<sup>2</sup> and the resistivity must be in  $\Omega$ mm<sup>2</sup>. You could always use m<sup>2</sup> at which point everything else would need to be converted to m<sup>2</sup>. Follow the example below and you will see what is meant.

1/. A length of copper conductor 55 m long, having an area of 2.5 mm<sup>2</sup> has a specific resistance of  $1.724 \times 10^{-8}$   $\Omega$ m. What is its resistance? What will happen if the temperature falls to 0 °C?

Remember it is always best to decide beforehand whether you are going to convert everything to metres or to millimetres. In this instance, I will change everything to metres and m<sup>2</sup>.

$$R = \frac{\rho l}{A}$$
  
(a) 20°C 
$$R = \frac{1.724 \times 10^{-8} \times 55}{2.5 \times 10^{-6}}$$
  

$$R = \frac{0.9482 \times 10^{-6}}{2.5 \times 10^{-6}}$$
  

$$R = \frac{0.379\Omega}{2.5 \times 10^{-6}}$$
  
(a) 0°C 
$$R = \frac{1.588 \times 10^{-8} \times 55}{2.5 \times 10^{-6}}$$
  

$$R = \frac{0.8734 \times 10^{-6}}{2.5 \times 10^{-6}}$$
  

$$R = 0.349\Omega$$

Notice that when  $mm^2$  are converted to  $m^2$  you are always able to write  $\times 10^{-6}$  next to the number. You should also notice that the resistance falls when the temperature lowers.

The other thing to remember is that when you are using your calculator it is unnecessary for you to type in the × (times) sign in  $2.5 \times 10^{-6}$ . On your scientific calculator you should have either an '*exp*' button or an '*ee*' button. This automatically puts in the 10.

For the typing in of  $1 \times 10^{-6}$ , and others of this type:-

- 1/. Press 1 on your calculator;
- 2/. Press **exp** or **ee**. You should see 1<sup>00</sup> in your calculator.
- 3/. Press the  $\frac{1}{2}$  key and then the number 6. You should see  $1^{-06}$ , or something like it.

If you look back in your **basic maths** study book, you should see similar exercises. If you are not sure, look back over your notes.

Try this next one.

2/. Compare the resistance of two conductors, one copper the other aluminium. The area of each is 120 mm<sup>2</sup> and the length of run is 125 m. Assume that the resistivity of copper is  $1.724 \times 10^{-8} \Omega m$  and aluminium is  $2.83 \times 10^{-8} \Omega m$ .

Again, make sure that everything is converted into the same units. I will convert the area again.

$$R = \frac{\rho I}{A}$$
Copper  $R = \frac{1.724 \times 10^{-8} \times 125}{120 \times 10^{-6}}$ 
Copper  $R = \frac{2.155 \times 10^{-6}}{120 \times 10^{-6}} = \underline{0.018\Omega}$ 
Aluminium  $R = \frac{2.83 \times 10^{-8} \times 125}{120 \times 10^{-6}}$ 
Aluminium  $R = \frac{3.5375 \times 10^{-6}}{120 \times 10^{-6}} = \underline{0.0295\Omega}$ 

Notice again the conversion of the area to m<sup>2</sup> so that it is in the same units as length and resistivity. This is very important. If you get this wrong, you will generally be 1 000 out one way or the other.

With this in mind let's just take a few moments to consider changing between different values of area and volume.

The idea of  $mm^2$  or even  $m^2$  may seem to be confusing. We are not squaring the answer we are merely expressing the answer in the units of  $mm^2$  or  $m^2$ . Have a look at the diagram below.



Not drawn to scale

#### Figure 5 Area differences

The table below shows some of the most common conversions. Become familiar with them.

Conversion	Multiply/Divide	Standard form
mm⇒m	÷1 000	1×10 <sup>-3</sup>
m⇒mm	×1 000	1×10 <sup>3</sup>
mm²⇒m²	÷1 000 000	1×10 <sup>-6</sup>
m²⇒mm²	×1 000 000	1×10 <sup>6</sup>
mm⇒cm	÷10	1×10 <sup>-1</sup>
cm⇒mm	×10	1×10 <sup>1</sup>
cm⇒m	÷100	1×10 <sup>-2</sup>
m⇒cm	×100	1×10 <sup>2</sup>
cm <sup>2</sup> ⇒m <sup>2</sup>	÷10 000	1×10 <sup>-4</sup>
mm³⇒m³	÷1 000 000 000	1×10 <sup>-9</sup>
m³⇒mm³	×1 000 000 000	1×10 <sup>9</sup>
$cm^3 \Rightarrow m^3$	÷1 000 000	1×10 <sup>-6</sup>
m³⇒cm <sup>3</sup>	×1 000 000	1×10 <sup>6</sup>

You need to become very familiar with all of these. Any slip in your working out and you can be over 1000 times out.

Try an example.

1/. A rectangle has sides of 150 mm by 3.2 m. What is its area?

A = lb  $A = 150 \times 3200 = \underline{480000 mm^2}$ Alternatively  $A = 0.15 \times 3.2 = \underline{0.48m^2}$ 

You should have noticed that I have either converted everything to metres or to millimetres. You must have common units. You cannot have a combination of metres and centimetres etc. you may also notice in this example that the difference between mm<sup>2</sup> and m<sup>2</sup> is exactly 1 000 000 (1 million) times.

#### Summary

Resistance is a function of a material, its length, area and temperature.

Although we have not considered temperature we recognise that an increase in temperature will cause the resistance to rise.

Ohm's law relates resistance to both voltage and current by stating that current and voltage are directly proportional to one another – as voltage increases so does current.

#### Exercise 1.

- A coil has an effective length of 12 m. What will be the resistance of the coil if its area is
   0.25 mm<sup>2</sup>. Assume the resistivity is taken from the table in this session?
- 2) If the length of a conductor doubles and the area doubles, what will happen to the resistance?
- 3) A conductor of length 40 m is extended to 80 m with no increase in size. If the initial resistance is  $1.2 \Omega$  what will be the new resistance of the extended cable?
- A coil is made up of 10 000 turns of copper wire. The diameter of the former around which the wire is wound is 39.8 mm, and the area of the conductor is 0.275 mm<sup>2</sup>. Calculate the resistance of the coil at 20 °C. [Hint: remember how to calculate the circumference of a circle]
- The light in a car park is supplied via a cable 85 m long. The area of the cable is 6 mm<sup>2</sup>.
   Calculate the resistance of the cable.

U	230		110	400		230	230	50
I	12	14		63	3	5		10
R		15	77		65		9	

6) Using just Ohm's Law fill out the table below.

7) For the following cable sizes calculate the resistance per metre of the conductors assuming that at 20 °C the resistivity of copper is  $1.724 \times 10^{-8}$  Ωm. After you have calculated your answers check them against the values for mV/A/m (effectively Ω/m) found in Appendix 4 of BS 7671 Table 4D1B and give reasons why your answers are different.

Size (mm <sup>2</sup> )	Answer
1.5	
2.5	
4.0	
6.0	
10.0	
16.0	
25.0	

If you have gone wrong, go back over the examples shown and redo the tables. Learn from your mistakes!

# Session 2

### Series circuits

By the end of this session you will have had the opportunity to:

• understand how resistors are added up when they are connected in series.

Up until now, all that we have considered has been a single load connected to a source. It is quite rare for circuits to be so simple, and the total load is often made up of a number of different resistors of one type or another. It is not so very difficult to come to terms with but certain principles do have to be taken on board.

#### Series circuits

When more than one resistor or load is connected end to end, then the resistors are said to be connected in series. You can see this from Figure 6.



Figure 6 Resistors connected in series

It would be worthwhile looking at this in more detail.

I will assume that we have a source or supply of 100 V and that the two resistors that are connected in the circuit are connected in series and have the same value, say 10  $\Omega$ .



#### Figure 7 Volt drop across resistors connected in series

What we could say is that the voltage that is dropped across each resistor would be exactly half the total supplied, that is 50 V would be dropped across each resistor.



Figure 8 Current in a series circuit

What we can also see is that the current must be the same at all points of the circuit. That is the current at point (**A**) is the same as at point (**B**) and at (**C**).

Looking at this more mathematically we can say that the total voltage supplied is the sum of the voltages dropped across the resistors, or  $U_{\text{Total}} = U_1 + U_2$ .

Now as you all so very familiar with Ohm's Law, you will be able to recognise that U = IR. What we can do is substitute IR for U every time we see it.

From this we get:-

$$U_{\text{Total}} = U_1 + U_2$$

$$IR_{\text{Total}} = IR_1 + IR_2$$

$$IR_{\text{Total}} = I(R_1 + R_2)$$

$$\int R_{\text{Total}} = \int (R_1 + R_2) \quad \text{current cancels as it is constant}$$

$$R_{\text{Total}} = R_1 + R_2$$

$$R_{\text{T}} = R_1 + R_2 + \dots + R_n$$

It may seem complicated but it is important for you to know that because the current is constant it can be cancelled and so things can be simplified. The equation that you have to remember is the last one.

$$R_{\rm T} = R_1 + R_2 + \dots + R_r$$

The more resistors connected the longer the equation will become.  $\mathbf{R}_n$  just stands for any number of resistors. We'll look at a couple of examples.

- 1/. Three lamps are connected in series. They have resistance values of  $20 \Omega$  each. If the supply voltage is 230 V what will be:
  - i). Total resistance;
  - ii). Total current;
  - iii). Voltage dropped across each resistor.

Always try to draw a diagram. It is invaluable in giving you an idea of what you are looking for. Remember it is a thumbnail sketch; do not spend too long on it.



There is a lot here so take your time in following it. The first section deals with adding up the resistors. The second section finds the current using our old friend Ohm's Law. The third section finds the voltages dropped across the loads, again using Ohm's Law.

Determine the total resistance first

i/. 
$$R_{\rm T} = R_1 + R_2 + R_3$$
  
 $R_{\rm T} = 20 + 20 + 20 = \underline{600}$ 

From this we can now find the total current flow

ii/. 
$$U = IR$$
 transpose

$$I = \frac{U}{R}$$
$$I = \frac{230}{60} = \underline{3.83A}$$

Current is the same in all parts of a series circuit so we can now determine the volt drop across each resistor

iii/. 
$$U_1 = IR_1 = 3.83 \times 20 = \underline{76.6V}$$
  
 $U_2 = IR_2 = 3.83 \times 20 = \underline{76.6V}$   
 $U_3 = IR_3 = 3.83 \times 20 = \overline{76.6V}$ 

There are three effective questions here and it is important that you don't try to rush the answer. Show all your working and fill in your thumbnail sketch as you find out more.

- 2/. Three resistors are connected in series A, B, and C. A has a resistance of  $120 \Omega$ , B has a volt drop across it of 40 V when 1 A flows. The total resistance of the circuit is  $300 \Omega$ . Calculate:
  - i). Resistance of B and C
  - ii). Volt drop across A and C
  - iii). Total volt drop.

Certain principles should have been remembered by now. Firstly, in a series circuit, the current is constant, and so the current through (B) is the same as for (A) and (C). The second principle is to draw a thumbnail sketch.



The working out is shown below.

i/.

$$U = IR \quad \text{transpose}$$

$$R_B = \frac{U}{I}$$

$$R_B = \frac{40}{1} = \underline{40\Omega}$$

$$R_T = 300\Omega$$

$$R_T = R_A + R_B + R_C \quad \text{transpose}$$

$$\therefore R_C = R_T - R_A - R_B$$

$$R_C = 300 - 120 - 40 = \underline{140\Omega}$$

ii/. In this instance because we now know the resistance of each of the resistors and the current in the circuit, all we have to do is apply Ohm's Law. Remember it is vitally important that you have the first part to the question correct. If you haven't, all the rest will be wrong.

$$U_{\rm A} = IR_{\rm A} = 1 \times 120 = \underline{120V}$$
$$U_{\rm C} = IR_{\rm C} = 1 \times 140 = \underline{140V}$$

- iii/. You can choose two ways in which to answer this last part.
  - Either recognise that both the total current and total resistance are known, at which point U = IR, or,
  - Add up the voltages that have been dropped across the loads.

Either way the answer will be the same. In fact, it is a check for you so that you can be sure that you have the right answer; the total supply voltage is always the sum of the other voltages.

$$U = IR$$
  

$$U = 1 \times 300 = \underline{300V} \text{ or}$$
  

$$U_{T} = U_{A} + U_{B} + U_{C}$$
  

$$U_{T} = 120 + 40 + 140 = \underline{300V}$$

I know it seems to take a lot of time and paper to come to the right answer, but it is important that you take your time and follow the procedure.

It can not be repeated often enough:

In a series circuit, the current is constant and never changes. In addition, the total voltage is the sum of all other voltages dropped across the loads or resistors.

### **Remember This!!**

#### Summary

So much learnt and so much more to learn!

Ohm's Law is a relationship between voltage and current assuming that temperature and material is constant-U = IR.

In a series circuit the current is constant.

In a series circuit  $R_T = R_1 + R_2 + \dots + R_n$ 

#### Exercise 2.

- 1) If a lamp has a resistance of 960  $\Omega$  and a supply voltage of 230 V. What will its current be?
- 2) What remains constant in a series circuit?
- 3) Two resistors of 7  $\Omega$  and 9  $\Omega$  are connected in series. If the supply voltage is 230 V, what will be its total current and the voltage dropped across each resistor?
- A motor draws 60 A from a distribution point 140 m away. The cable has a resistance of
   0.01 Ω per 100 m of single conductor. The voltage at the intake position is 425 V.
  - a). Draw a sketch of the circuit
  - b). Calculate the volt drop in the cable
  - c). Calculate the voltage at the motor.
- 5) An aluminium cable, 350 m long, has two aluminium conductors, each having a crosssectional area of 95 mm<sup>2</sup>. A current of 124 A is drawn by the supply. Calculate:
  - a). Volt drop in the cable if the resistivity is assumed to be  $2.83 \times 10^{-8} \Omega m$ ;
  - b). The voltage at the load if the supply is 245 V.

Now move on to session 3.

# Session 3

## Parallel circuits

By the end of this session you will have had the opportunity to:

• understand how resistors connected in parallel are different to series connected resistors.

In the previous session we considered the nature of resistors connected in series and saw that current is constant in a series circuit and that the voltage supplied to the circuit is equal to the sum of the volts dropped across each of the resistors. This is a classic case of Kirchhoff's Voltage Law:

#### In a closed loop, the algebraic sum of all voltages is zero.

Or in other words:

#### The supply voltage equals all other volt drops added together in a circuit.

In the work we will be revising with parallel circuits there are some changes.

#### Parallel circuits

A parallel circuit is one where all the resistors have the same start and finish point. Look at Figure 9.



#### Figure 9 Parallel circuit

You can see that the supply is connected to the ends of each resistor. The first important principle of parallel circuits can be seen here, and that is the voltage supplied to each resistor in a parallel network, irrespective of the size of resistor is the same.

What happens to the current in a parallel circuit? What you must remember is that the current flowing into a point must equal the current flowing away from a point.

This makes very good sense. In the diagram over the page, you can see this principle at work. The current supplied comes to a joint or junction, there are only two ways for it to travel, and it must therefore divide down the two paths available to it.



Figure 10 Current flow in a parallel circuit

In a parallel circuit, the voltage dropped across the loads is constant as the source is connected across the loads in exactly the same way, whilst it is the current that divides. The total current supplied is the sum of the currents in each of the legs of the network, or  $I_{Total} = I_1 + I_2$ . This is opposite to what happens with a series circuit.

As with the series circuits we can substitute Ohm's Law,  $I = \frac{U}{R}$  in the above equation. What we get is this:

$$I_{T} = I_{1} + I_{2}$$

$$I = \frac{U}{R} \quad \text{substitute}$$

$$\frac{U}{R_{T}} = \frac{U}{R_{1}} + \frac{U}{R_{2}}$$

$$U \frac{1}{R_{T}} = U \left( \frac{1}{R_{1}} + \frac{1}{R_{2}} \right)$$

$$U \frac{1}{R_{T}} = \mathcal{I} \left( \frac{1}{R_{1}} + \frac{1}{R_{2}} \right)$$

$$\frac{1}{R_{T}} = \frac{1}{R_{1}} + \frac{1}{R_{2}} + \dots + \frac{1}{R_{n}}$$

Here,  $\frac{U}{R}$  is substituted for *I* whenever it is seen. Because the voltage is constant it can be cancelled out and so *U* becomes 1.

The part of the equation to remember is the last line. The  $R_n$  means that this is true for any number of resistors in parallel.

$$\frac{1}{R_{\rm T}} = \frac{1}{R_{\rm 1}} + \frac{1}{R_{\rm 2}} + \frac{1}{R_{\rm 3}} + \dots + \frac{1}{R_{\rm n}}$$

Some examples should help you to become familiar with the ideas and principles set out here.

- 1/. Resistors of  $8 \Omega$  and  $10 \Omega$  are connected in parallel. If the supply voltage is 110 V, calculate:
  - i/. Total resistance
  - ii/. Total current
  - iii/. Current in each resistor.

Remember to draw a small thumbnail sketch, with as much detail as possible on it.



i/.

$$\frac{1}{R_{\rm T}} = \frac{1}{R_{\rm 1}} + \frac{1}{R_{\rm 2}}$$
$$\frac{1}{R_{\rm T}} = \frac{1}{8} + \frac{1}{10}$$
$$\frac{1}{R_{\rm T}} = 0.125 + 0.1 = 0.225$$
$$R_{\rm T} = \frac{1}{0.225} = 4.44\Omega$$

Notice that you cannot just turn the equation upside down. Notice also that it is a question of doing fractions.

Another way of determining the total resistance of a parallel circuit could be:

If you have a basic scientific calculator, it should have a  $\frac{1}{x}$  or a  $x^{-1}$  button. Follow the following procedure every time and you will get the correct answer.

- 1/. Type in the first number, in this instance **8**.
- 2/. Press  $\frac{1}{2}$ . You should read 0.125 in this instance.
- 3/. Press +.
- 4/. Type in the next number, in this case **10**.
- 5/. Press  $\frac{1}{2}$ . You should read 0.1 this time.
- 6/. Press =. You should read 0.225.
- 7/. Press  $\frac{1}{x}$ . You should read 4.4444444 etc.

This is the final answer. By all means use this method, but more importantly know what you are doing, do not think the calculator will get you out of trouble, it won't!

We are now looking for the total current.

ii/.

$$U = IR \text{ transpose}$$
$$I = \frac{U}{R}$$
$$I = \frac{110}{4.44} = \underline{24.75A}$$

Once the resistance has been found it is merely the application of Ohm's Law. Remember to fill in your thumbnail sketch with more detail as you find more out.

iii/. In this part, what you have to remember is that the voltage is constant. All we need to do is apply Ohm's Law twice; once for each of the resistors.

$$U = IR \quad \text{transpose}$$
$$I = \frac{U}{R}$$
$$I_8 = \frac{110}{8} = \underline{13.75A}$$
$$I_{10} = \underline{110}$$
$$10 = \underline{114}$$

Notice that if you add up the two currents they equal the total current supplied, this is a useful check. If they don't add up then you have gone wrong somewhere so go back and check!

It is probably worth while considering something that you may come across in parallel circuit work.

#### Product over sum

In a two-resistor parallel circuit we can simplify how we set this out to make life a little easier when using our calculators.

$$\frac{1}{R_{T}} = \frac{1}{R_{1}} + \frac{1}{R_{2}}$$

$$\frac{1}{R_{T}} = \frac{1}{R_{1}R_{2}}$$
find the lowest common denominator
$$\frac{1}{R_{T}} = \frac{R_{2} + R_{1}}{R_{1}R_{2}}$$
carry out a normal fractions calculation
$$R_{T} = \frac{R_{1}R_{2}}{R_{1} + R_{2}}$$
invert

This is a standard adding together of fractions.

What we have then is that in a two-resistor network we can use this product over sum rule and speed up our calculations. In our first example this would appear as:-

$$\frac{1}{R_{\rm T}} = \frac{1}{R_{\rm 1}} + \frac{1}{R_{\rm 2}}$$
$$R_{\rm T} = \frac{R_{\rm 1}R_{\rm 2}}{R_{\rm 1} + R_{\rm 2}} = \frac{8 \times 10}{8 + 10}$$
$$R_{\rm T} = \frac{80}{18} = \underline{4.44\Omega}$$

Another example will be helpful.

- 2/. Three resistors of 6  $\Omega$ , 9  $\Omega$  and 12  $\Omega$  are connected in parallel. A current of 2 A is flowing in the 12  $\Omega$ . Calculate:
  - i/. Supply voltage
  - ii/. Current in each resistor leg
  - iii/. Total current
  - iv/. Total resistance.

Remember to draw a thumbnail sketch.



Here what you have to remember is that the voltage dropped across any one resistor in the parallel network is the same for all resistors in the network. So because the  $12 \Omega$  resistor has 2 A flowing through it we can work out the voltage dropped across that resistor. This means we will have worked out the voltage across all three.

I will not explain everything this time; just see if you can follow the working out.

Find the total voltage

i/. 
$$U = IR$$
  
 $U = 2 \times 12 = \underline{24V}$ 

Find the current in each leg

ii/. 
$$I = \frac{U}{R}$$
  
 $I_6 = \frac{24}{6} = \underline{4A}$   
 $I_9 = \frac{24}{9} = \underline{2.67A}$ 

Find the total current

iii/. 
$$I_{\rm T} = \frac{U}{R}$$
 we don't know *R* yet  
So  $I_{\rm T} = I_6 + I_9 + I_{12} = 2.67 + 4 + 2 = 8.67A$ 

Find the total resistance

iv/. 
$$\frac{1}{R_{T}} = \frac{1}{R_{1}} + \frac{1}{R_{2}} + \frac{1}{R_{3}}$$
$$\frac{1}{R_{T}} = \frac{1}{6} + \frac{1}{9} + \frac{1}{12}$$
$$\frac{1}{R_{T}} = 0.166 + 0.111 + 0.083 = 0.361$$
$$R_{T} = \frac{1}{0.361} = 2.77\Omega$$

I know that this seems very complex, but notice that Ohm's Law is very prominent once again. Take things step by step and don't worry too much about where you start. In many instances we can have different starting positions for the problem and the answers will still drop out OK.

We could calculate the total resistance by using the product over sum rule twice.

$$\frac{1}{R_{\rm T}} = \frac{1}{R_{\rm 1}} + \frac{1}{R_{\rm 2}} + \frac{1}{R_{\rm 3}}$$
$$R_{\rm T} = \frac{R_{\rm 1}R_{\rm 2}}{R_{\rm 1} + R_{\rm 2}} = \frac{6 \times 9}{6 + 9} = \frac{54}{15} = \underline{3.6\Omega}$$

Now repeat for the remaining resistor

$$R_{\tau} = \frac{R_1 R_2}{R_1 + R_2} = \frac{3.6 \times 12}{3.6 + 12} = \frac{43.2}{15.6} = \underbrace{2.77\Omega}_{-1.00}$$

Notice that the numbers come out the same.

#### Summary

So much learnt and so much more to learn!

In a parallel circuit the voltage dropped across individual resistors is the same.

In a parallel circuit 
$$\frac{1}{R_{T}} = \frac{1}{R_{1}} + \frac{1}{R_{2}} + \frac{1}{R_{3}} + \dots + \frac{1}{R_{n}}$$

The product over sum rule can be used in a two-resistor network,  $R_T = \frac{R_1 R_2}{R_1 + R_2}$ 

A parallel circuit is a classic application of Kirchhoff's Current Law. Current flowing into a node (joint) must equal the current flowing from that point.

#### Exercise 3.

- 1) Two resistors of 7  $\Omega$  and 9  $\Omega$  are connected in parallel. If the supply voltage is 230 V, what will be the total resistance, the total current and the current in each leg?
- 2) Three resistors each of 8  $\Omega$  are connected:
  - a). In series
  - b). In parallel

Find out anything you can about each of the circuits, if they are supplied from a 110 V source.

- 3) A load draws a current of 220 A through a cable having a resistance of 0.03  $\Omega$ /100 m of single conductor. The length of run is 230 m.
  - a). What will be the total resistance of the cable if a second cable of the same size is connected in parallel with the first?
  - b). What will be the new voltage at the load if the supply voltage is 245 V?
- Two loads, A and B draw currents of 25 A and 38 A respectively. The resistance of the two-wire cable is 0.12 Ω/100 m twin cable. The cable runs for 70 m from the distribution board to the first load and then a further 24 m to the next load. Determine:
  - a). The current drawn in each leg of the circuit
  - b). The resistance of each part of the circuit [Not the loads]
  - c). The volt drops in each leg of the circuit.

# Session

### Power and energy

By the end of this session you will have had the opportunity to:

- describe the relationship between power and energy
- determine how power and energy may be calculated.

In this session we will consider the nature of power and energy. You will have probably covered both power and energy at GCSE level science; however we need to spend some time clarifying those areas.

#### Energy

In many ways energy is a difficult concept to lay hold of, but a simple definition could be:-

#### The ability to do work.

Energy can be considered in a range of areas including mechanical, thermal and electrical. The common feature is that work is either being done or is capable of being done.

Energy or work done is measured in *joules (J)*. This unit is named after James P. Joule who carried out work on the relationship between mechanical and thermal energies. The joule itself is defined as:

The work done when a force of 1 newton is exerted through a distance of 1m in the direction of the force.

In effect this makes energy a measure of the work done and can be calculated in a number of ways depending on the nature of the energy.

Make a list of electrical or mechanical devices that store or transmit energy.



In a thermal system the energy stored is dependent on the nature of the material. The heat required to raise the temperature of a substance of 1 kg by 1 °C is called the *specific heat capacity* (*shc*).

Thermal energy is calculated using:-

$$Q = mc\Delta T$$
$$Q = mc(T_2 - T_1)$$

Where:

Q = quantity of heat (energy) in joules

m = mass in kg

c = specific heat capacity of a particular substance in joules/kg°K

 $\Delta T$  = change in temperature in K (kelvin)

The table below provides some values of specific heat capacity.

Substance	Specific heat capacity J/kgºK
Water	4190
Air	1015
Aluminium	960
Copper	380
Iron	420
Mercury	140
Nichrome	430
Silver	230

You can see from this that water is very good for storing heat as it requires a lot of energy (over 4 000 J) to raise the temperature by 1 °C; even air requires around 1 000 J of energy to raise the temperature by 1 °C, whereas the metals all require less energy to heat.



Figure 11 Heat delivered to a substance

In Figure 11 heat is delivered to a substance and the temperature of the substance will rise.

In Figure 12 the current flow in the conductor causes its temperature to rise. When the current causes the temperature of the cable to rise to a greater level than the surrounding air or substance then heat is given off.



Figure 12 Temperature rise in a current carrying conductor

A container 2 m long by 1.5 m wide by 1.2 m high is filled with water. The water has a temperature of 7  $\degree$ C and has to be raised to a temperature of 45  $\degree$ C. The mass of 1 litre of water is assumed to be 1 kg. If the specific heat capacity of water is 4200 J/kgK determine the energy required assuming there are no losses from the container.



[Answer: 574560 J or 574.56 kJ]

#### Mechanical energy

With mechanical energy we have to consider two different forms:

- Kinetic
- Potential.

#### **Potential energy**

Consider Figure 13, which has an object (great lump) at the top of a cliff.



#### Figure 13 Potential energy

The amount of energy available is dependent on the mass of the object at the top of the cliff, the height that the object is above the ground and the acceleration due to gravity. So:

P.E = mgh

Where:

P.E = potential energy in J

m = mass in kg

$$g =$$
 acceleration due to gravity in m/s<sup>2</sup>

h = height in m

It is assumed that the acceleration due to gravity is constant at 9.81 m/s<sup>2</sup>, although this figure does vary at different points on the earth's surface.

#### **Kinetic energy**

Kinetic energy is energy related to movement. Now consider Figure 14.



Figure 14 Kinetic energy

The potential energy at the top of the cliff is now being converted into kinetic energy as the object falls. The kinetic energy can be determined using:-

$$K.E = \frac{1}{2}mv^2 = \frac{mv^2}{2}$$

Where:

K.E = kinetic energy in J

m = mass in kg

v = velocity of the object in m/s

When the object hits the ground 'work' is done on the ground, as well as on the internal organs of the object. Eventually all energy turns to heat and heat is given off.

1) An object of mass 80 kg is set on a pylon of height 30 m. What is the potential energy of the object assuming that the acceleration due to gravity is 9.81 m/s<sup>2</sup>?

2) Assuming that there are no losses due to friction, at what velocity will the object hit the ground?

ANSWER 1) 23544 J or 23.54 kJ 2) 24.26 m/s.

#### **Electrical energy**

As with thermal and mechanical energy, electrical energy is measured in joules (J), and as with the other two forms of energy the work done is dependent on a number of elements. With thermal energy we have to consider the mass, specific heat capacity and the temperature of the substance. With mechanical energy we are dealing with mass, velocity, height and acceleration due to gravity depending on whether we are looking at kinetic or potential energy. With electrical energy the elements that we consider depend on the flow of current and the voltage that is dropped across a particular component.

So:

$$W = IUt$$

Where:

W = electrical energy in J I = current in A U = voltage in V

t = times in s



Figure 15 Electrical energy converted to heat within a current carrying conductor

As the current flows heat is given off, with time the amount of heat given off will increase.

There are other forms of electrical energy related to capacitance and inductance, however for the present we will simply consider resistive loads.

An electrical load of resistance  $10 \Omega$  is connected across a supply voltage of 230 V. If the load were to be connected for a period of 33 minutes, what would be the energy dissipated by the load?

ANSWER 10474200 J or 10.47 MJ

#### Power

Power is closely related to energy. If energy is the work done on a system, an object or a substance, then power is defined as:

The rate at which work is done	
Or	
The rate at which energy is converted.	

In effect, with power we are more concerned with how quickly the heat is given off by the load.



Figure 16 Low power output



Figure 17 High power output

The larger the voltage and current, the greater will be the power output and this can be demonstrated by:

P = IU

Where:

P = power in W

*l* = current in A

U = voltage in V

Power, in whatever system we deal with is measured in watts (W) and:

1 watt = 1 Joule/second  $\binom{J}{s}$ 

When we look at the power formula on the previous page we can see that it makes use of both voltage and current. We can apply elements of Ohm's law to provide ourselves with some variations on a theme.

$$P = IU \text{ and } U = IR$$

$$P = I(IR) = I^{2}R$$

$$P = IU \text{ and } I = \frac{U}{R}$$

$$P = \left(\frac{U}{R}\right)U = \frac{U^{2}}{R}$$

$$\therefore P = \underline{IU} = \frac{U^{2}}{\underline{R}} = \underline{I^{2}R}(W)$$

All we now have to do is to start using them. Let's consider an example from an earlier session.

- 1/. Three resistors of 6  $\Omega$ , 9  $\Omega$  and 12  $\Omega$  are connected in parallel. A current of 2 A is flowing in the 12  $\Omega$ . Calculate:
  - i/. Supply voltage
  - ii/. Current in each resistor leg
  - iii/. Total current
  - iv/. Total resistance.

Remember to draw a thumbnail sketch.



We saw that in this circuit the current divided down each leg of the circuit, but that the voltage remained the same across each resistor. Below you can see a copy of the working out that we did.

Find the total voltage  
i/. 
$$U = IR$$
  
 $U = 2 \times 12 = \underline{24V}$   
Find the current in each leg  
ii/.  $I = \frac{U}{R}$   
 $I_6 = \frac{24}{6} = \underline{4A}$   
 $I_9 = \frac{24}{9} = \underline{2.67A}$   
Find the total current  
iii/.  $I_T = \frac{U}{R}$  we don't know *R* yet  
So  $I_T = I_6 + I_9 + I_{12} = 2.67 + 4 + 2 = \underline{8.67A}$   
Find the total resistance  
 $iV/. \quad \frac{1}{R} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R}$ 

$$R_{T} = \frac{R_{1} + R_{2} + R_{3}}{R_{T}}$$
$$\frac{1}{R_{T}} = \frac{1}{6} + \frac{1}{9} + \frac{1}{12}$$
$$\frac{1}{R_{T}} = 0.166 + 0.111 + 0.083 = 0.361$$
$$R_{T} = \frac{1}{0.361} = 2.77\Omega$$

Now we'll consider the nature of the power loss in each resistor and in the circuit as a whole.

Total voltage 
$$U = \underline{24V}$$
  
Current in each leg  $I = I_6 = \underline{4A}$   $I_9 = \underline{2.67A}$   $I_{12} = \underline{2A}$   
Total current  $I_T = 2.67 + 4 + 2 = \underline{8.67A}$   
Total resistance  $\frac{1}{R_T} = \frac{1}{6} + \frac{1}{9} + \frac{1}{12} = \underline{2.77\Omega}$   
 $P = I^2 R$   
 $P_6 = 4^2 \times 6 = \underline{96W}$   
 $P_9 = 2.67^2 \times 9 = \underline{64.16W}$   
 $P_{12} = 2^2 \times 12 = \underline{48W}$   
 $P_T = 8.67^2 \times 2.77 = \underline{208.2W}$   
 $P_T = P_6 + P_9 + P_{12}$   
 $208.2 = 96 + 64.16 + 48$ 

Use two other power formulae and check that the answers work out the same.

#### Summary

Energy is defined in terms of the work that can or is being done.

There are a variety of forms that energy possesses in terms of thermal, mechanical and electrical. In thermal systems energy is measured in terms of the need to raise or lower a temperature of a substance.

In mechanical systems potential energy is to be seen as stored or untapped energy, whilst kinetic energy is related to movement.

In electrical systems energy is measured in terms of the time that current and voltage are supplied to a system.

Power is defined as the rate at which work is done.

Power in a resistive electrical system can be determined in three ways:-

$$P = \underline{IU} = \frac{U^2}{\underline{R}} = \underline{I^2 R}(W)$$

#### Exercise 4.

- 1) If a lamp has a resistance of 960  $\Omega$  and a supply voltage of 230 V what will its power rating?
- 2) Two resistors of 7  $\Omega$  and 9  $\Omega$  are connected in series. If the supply voltage is 230 V, what will be the total power dissipated and the power dissipated from each resistor?
- A motor draws 60 A from a distribution point 140 m away. The cable has a resistance of 0.01 Ω/m per 100 m of single conductor. The voltage at the intake position is 425 V.
   What will be the power dissipated in the load and in the cables?
- 4) An aluminium cable, 350 m long, has two aluminium conductors, each having a cross-sectional area of 95 mm<sup>2</sup>. A current of 124 A is drawn by the supply. Calculate:
  - a) Voltage drop in the cable if the resistivity is assumed to be  $2.83 \times 10^{-8} \Omega m$
  - b) The voltage at the load if the supply is 245 V
  - c) The power dissipated in the cable, the total power available, and the power available at the load.

- 5) Two resistors of 7  $\Omega$  and 9  $\Omega$  are connected in parallel. If the supply voltage is 230 V, what will be the total power dissipated and the power dissipated in each resistor?
- 6) A load draws a current of 220 A through a cable having a resistance of 0.03  $\Omega$ /100 m of single conductor. The length of run is 230 m.
  - a) What will be the total resistance of the cable if a second cable of the same size is connected in parallel with the first?
  - b) What will be the voltage at the load if the supply voltage is 245 V?
  - c) What will be the power dissipated in the cable, and at the load?
- 8) Two loads, A and B draw currents of 25 A and 38 A respectively. The resistance of the two-wire cable is  $0.12 \Omega/100$  m twin cable. The cable runs for 70 m from the distribution board to the first load and then a further 24 m to the next load. Determine:
  - a) The current drawn in each leg of the circuit
  - b) The resistance of each part of the circuit [Not the loads]
  - c) The volt drops in each leg of the circuit
  - d) The power losses at all points as well as the power dissipated by the loads.

Now for the end exercise.

#### Appendix 1

There are a series of practical tasks that you should now have sufficient knowledge to be capable of attempting. Suggested experiments/practical work sessions would include:-

- 1). Measure voltage and current for resistors connected in series and parallel.
- 2). Use a wattmeter and an energy meter to measure the power dissipated in a circuit and the energy paid for?

These are just some suggested areas for practical activities that would be appropriate for you to understand.

Now try the last exercise.

#### **B&B Training Associates**

#### **Engineering Learning Materials**

Attempt all questions.

All marks are shown in the right-hand margin. You should aim to pass with a 85 % minimum mark. Anything less than this mark should lead you to re-read the text. 1/. What is the constant value in a series circuit? 1 What remains the same in a parallel circuit? 2/. 1 Two resistors of 4  $\Omega$  and 6  $\Omega$  are connected in series across a 72 V supply. 3/. What current is drawn and what is the volt drop across each resistor? 4 4/. Two resistors of 4  $\Omega$  and 6  $\Omega$  are connected in parallel across a 72 V supply. What current is drawn and what is the current flow through each resistor? 4 If the resistance of 800 m of a wire is 32  $\Omega$ , what would be the resistance of 5/. 250 m? 2 6/. The resistance of 575 m of a conductor is 21.85  $\Omega$ , how much is to be cut from it to make a resistance of 14.25  $\Omega$ ? 2 7/. Using the specific resistance values quoted in the text determine the resistance of 40 m of copper of area 1.5 mm<sup>2</sup> at a temperature of 20 °C. 4 What will be the diameter of 1 000 m of copper cable whose resistance is 8/. 19.09  $\Omega$  at a temperature of 0 °C 5 9/. A copper coil has a resistance of 0.43  $\Omega$  at 20 °C. The coil of wire is wrapped around a former of diameter 25 mm. The coil has 200 turns on it. What is the diameter of the coil? 6 10/. An electric lamp draws 9.6 A at 50 V. It is operated from a 110 V supply. Find the resistance value required to limit the current drawn to this value. 4 11/. Three resistors of 8.4  $\Omega$ , 6.8  $\Omega$  and 4.8  $\Omega$  are connected in series across a 110 V supply. Determine the total resistance, current and the volt drop across each resistor. 5 12/. For Q.3 determine the power dissipated and the energy consumed over a 23minute period for each resistor. 4 For Q.4 determine the power dissipated and the energy consumed over a 13/. 23minute period for each resistor. 4 For Q.11 determine the power dissipated and the energy consumed over a 14/. 3minute period for each resistor. 4

4

8

- 15/. A coil of resistance 4  $\Omega$  is connected across a shunt of resistance 0.005  $\Omega$ . If the current in the main circuit is 50 A, find the current in the coil.
- 16/. For the circuit shown below determine the value of resistor  $\mathbf{R}_{\mathbf{x}}$  to two (2) significant figures.



17/. Determine the volt-drop across the 6  $\Omega$  resistor in the circuit below.



